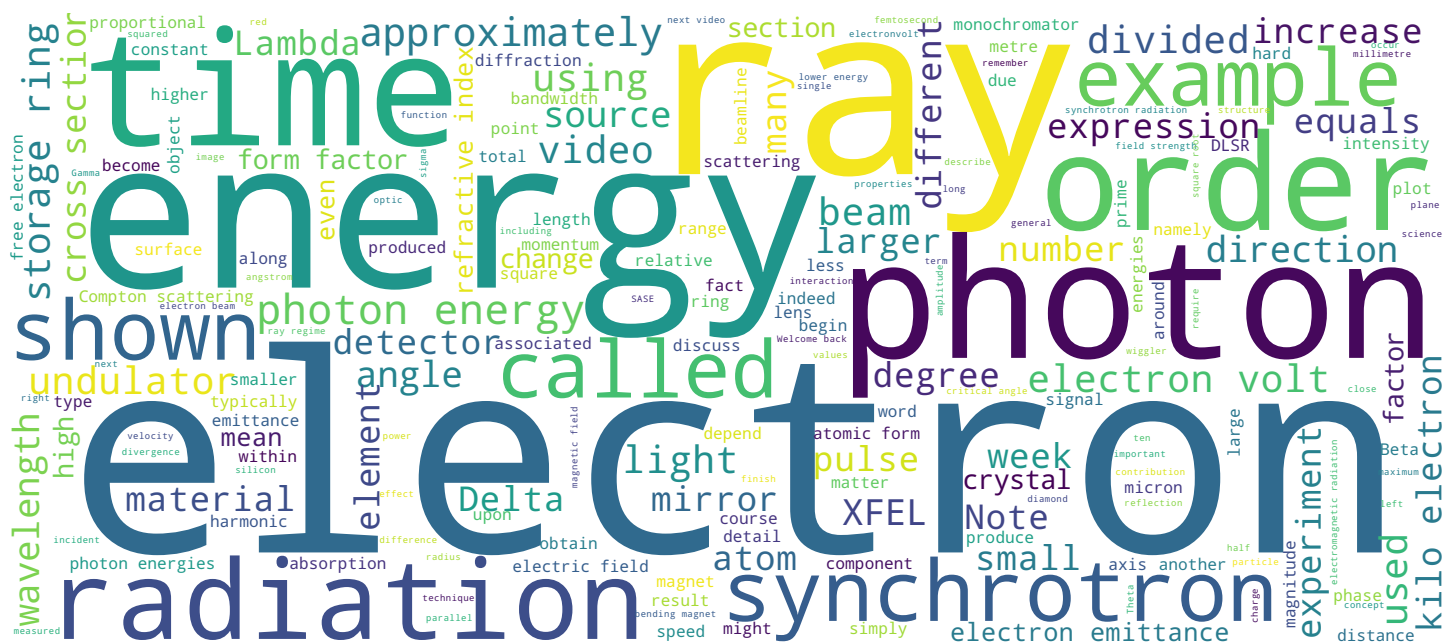


# Synchrotrons and x-ray free-electron lasers

## Techniques and applications

■ École polytechnique fédérale de Lausanne



# Contents and objectives of this video



- Scattering of high-energy photons
- Compton effect

In this video, we will discuss Compton scattering. This is a quantum mechanical phenomenon that requires the corpuscular photon description of light. Energy is transferred from the photon to the electron with the result that the scattered photon has a lower energy than the incident radiation. Compton scattering is used to obtain information about the electronic structure of condensed matter. The Compton cross section is thus used as a probe to measure the electronic momentum distribution. It is most commonly used at facilities which can deliver very high energy photons, typically over 80 kilo electron volts.

Notes

Summary



0m 05s

# Quantum-mechanical description of photons



$$E = h\nu = \hbar\omega$$

$$p = h/\lambda = \hbar k$$

e.g.  $E = 100 \text{ keV}$

$\lambda = 1.24 \times 10^{-11} \text{ m}$

$p = 5.34 \times 10^{-23} \text{ kg m s}^{-1}$

10 keV electron:  $v \simeq (2E/m_e)^{1/2} = 5.95 \times 10^7 \text{ m s}^{-1}$   
 $p = (2m_e E)^{1/2} = 1.88 \times 10^7 \times 9.11 \times 10^{-31} = 5.4 \times 10^{-23} \text{ kg m s}^{-1}$

As we've already mentioned, the energy and momentum of a photon are given by  $E = h\nu$  and  $p = h/\lambda$ , which is equal to  $\hbar k$ . The momentum of a 100 kilo electron volt photon is approximately 5 times 10 to the minus 23 kilograms metres per second, which is approximately the same as the momentum of a 10 kilo electron volt electron.

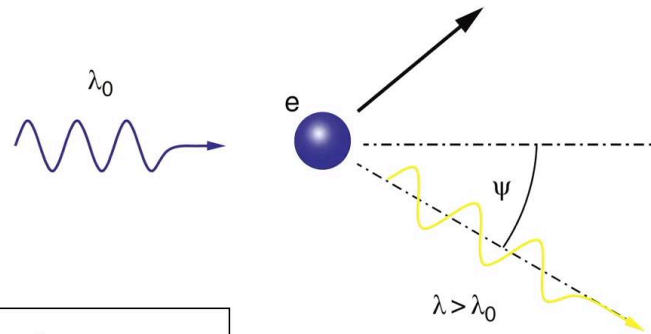
Notes

Summary



0m 51s

# Quantum-mechanical description of photons



Compton scattering equation

$$\lambda = \lambda_0 + \underbrace{\left[ \frac{h}{m_e c} \right]}_{\lambda_C = 2.426 \times 10^{-12} \text{ m}} (1 - \cos \psi)$$

Consider then, a photon with a wavelength  $\lambda_0$  incident on a stationary electron of mass  $m_e$ . Energy and momentum transfer occur between the photon and electron, meaning that the former is inelastically scattered through an angle  $\Psi$ , and with a lower energy, and hence longer wavelength. And the latter therefore recoils with a velocity  $v$ . We can apply the conservation of momentum and energy, leading to the two equations given here. After some straightforward but lengthy mathematical manipulation of these expressions, we arrive at the Compton scattering equation, namely, that the inelastically scattered photon has a wavelength  $\lambda$  equal to the incident wavelength, plus  $h / m_e c$  times 1 minus Cosine  $\Psi$ , where the constant  $h / m_e c$  is called the Compton wavelength and is equal to 2.426 picometres.

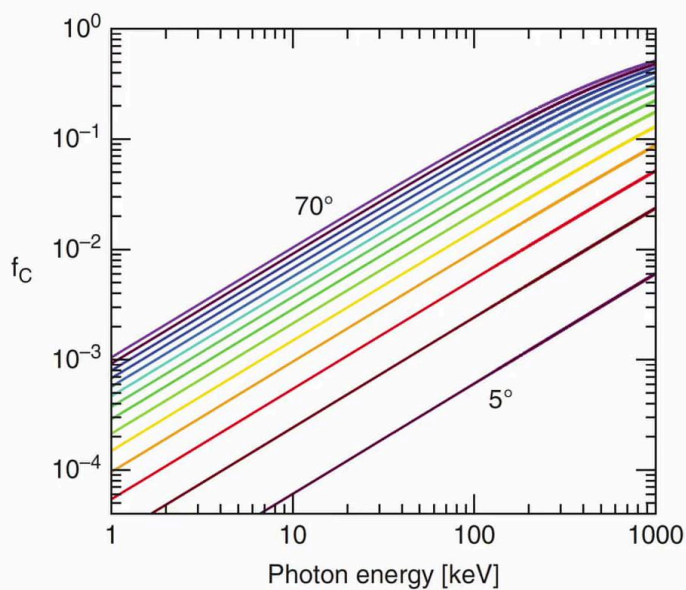
Notes

Summary



1m 20s

# Fractional loss in Compton scattering



$$f_C = \frac{h\nu_0 - h\nu}{h\nu_0}$$

$$= \frac{(E_0/m_e c^2)(1 - \cos \psi)}{1 + (E_0/m_e c^2)(1 - \cos \psi)}$$

The fractional loss in photon energy transferred to the electron is given by  $f_C$ , which is equal to  $h\nu_0 - h\nu / h\nu_0$ . This is plotted here for different photon scattering angles. Note that in the forward direction when  $\Psi$  is equal to 0, Cosine  $\Psi$  is 1, and so there is no energy transfer at all.

Notes

Summary

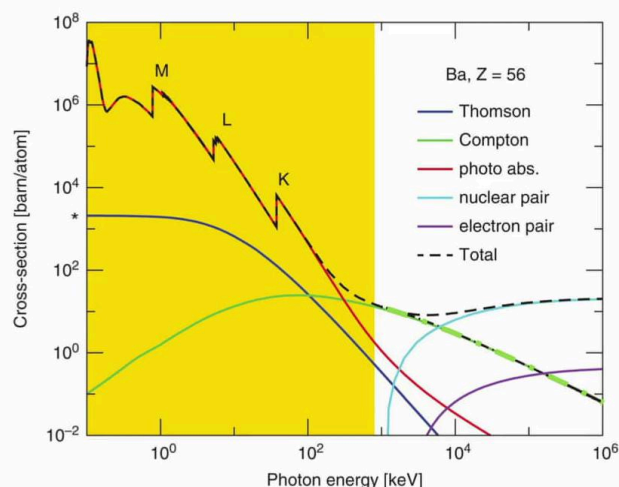


# Cross-section for Compton scattering

$$\sigma_C \approx 2Z\pi r_0^2 \left\{ \frac{1+\zeta}{\zeta^2} \left[ \frac{2(1+\zeta)}{1+2\zeta} - \frac{\ln(1+2\zeta)}{\zeta} \right] + \frac{\ln(1+2\zeta)}{2\zeta} - \frac{1+3\zeta}{(1+2\zeta)^2} \right\}$$

$$(\zeta = h\nu/m_e c^2)$$

$$\approx \frac{Z\pi r_0^2}{2\zeta} [1 + 2\ln(2\zeta)] \quad \zeta \gg 1$$



The cross section for incoherent Compton scattering is significantly smaller than that for coherent Thompson scattering up to several tens of kilo electron volt photon energy. For photon energies much larger than the rest mass energy of the electron of 511 kilo electron volts, the complex expression for the Compton cross section simplifies considerably to the expression highlighted here and shown by the green dot-dashed curve in the plot.

Notes

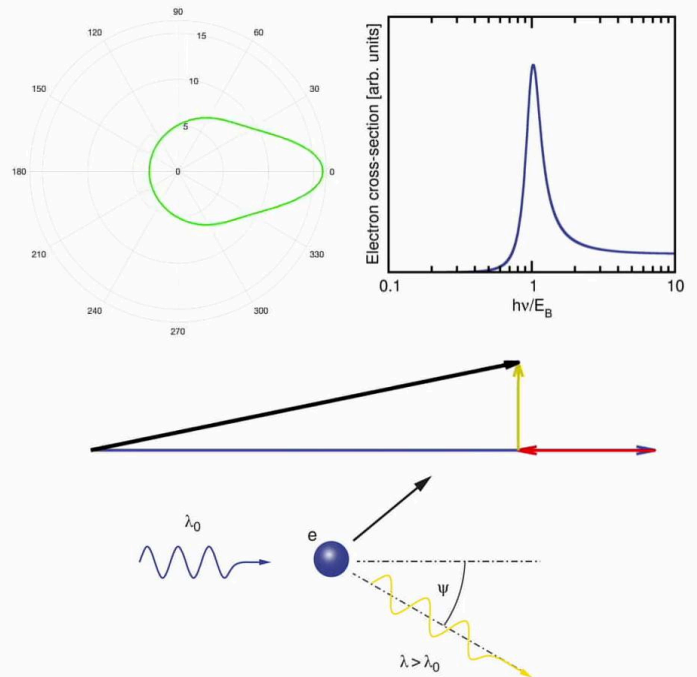
Summary



2m 57s



# Summary of this section



In summary, the three videos of this section have been concerned with the atomic form factor which describes the scattering properties of the elements. We initially looked at the angular and energy dependence of elastic scattering, not taking into consideration the fact that electrons in matter are generally bound to atoms. We then included this increased level of sophistication in the form of a damped oscillator model from which we arrived at the full atomic form factor. That, in addition to the slowly scattering vector dependent form factor, also included two energy dependent terms,  $f'$  and  $f''$ . Lastly, we also considered inelastic Compton scattering of photons by free electrons, which derives from the consideration of the conservation of energy and momentum when the electromagnetic radiation is described as collection of photons. In the last section of this week, we derive an expression for the complex refractive index of matter in the X ray regime, and use this to obtain expressions for reflection, refraction, and absorption. We will finish with a description of the processes that can occur after photo absorption. The knowledge gained here will be invaluable, not only for experimental methods such as X ray reflectivity and Xray absorption spectroscopies, but also for the design of many X ray optical elements, such as mirrors and lenses.

Notes

Summary



3m 33s